

amc 8 problems and solutions

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AMC 8 Problems and Solutions: A Complete Guide for Students and Teachers

Every year, thousands of middle school students across the United States and beyond prepare for the **American Mathematics Competitions 8 (AMC 8)**. This 25 minute, 25 question contest tests problem solving skills, creativity, and a solid grasp of elementary mathematics. While the exam is designed to be challenging, it is also an excellent way to sharpen logical thinking and build confidence in math. In this article, we'll dive into the structure of the AMC 8, discuss common pitfalls, and walk through a selection of representative problems with detailed solutions. Whether you're a student aiming for a top score or a teacher looking to support your class, this guide will give you the tools you need to succeed.

Understanding the AMC 8 Format

The AMC 8 is administered annually during the second week of March. It is divided into three sections, each with a specific focus:

1. Section 1 – Basic Algebra & Geometry (10 questions) – These questions test fundamental algebraic manipulation, simple equations, and basic geometric reasoning.
2. Section 2 – Counting & Probability (10 questions) – This section covers combinatorics, permutations, combinations, and basic probability concepts.
3. Section 3 – Advanced Geometry & Number Theory (5 questions) – The final section challenges students with more complex geometric constructions and number theory problems.

All questions are multiple choice, with five answer choices (A–E). Students can earn 6 points for a correct answer, 1.5 points for an unanswered question, and no penalty for incorrect answers. The maximum possible score is 150 points.

Common Mistakes and How to Avoid Them

While the problems may appear straightforward at first glance, many students fall into the following traps:

- Misreading the question – Carefully read every word; a single phrase can change the entire problem.
- Forgetting to consider all cases – Especially in combinatorics, overlooking a scenario can lead to an incorrect answer.
- Rushing calculations – A quick mental mistake can cost valuable points. Double check arithmetic.
- Overcomplicating simple problems – Sometimes the easiest approach is the best. Look for shortcuts.
- Skipping the “check” step – After solving, verify that the answer fits the context of the problem.

Sample Problems & Solutions

Below are ten carefully selected AMC 8 problems that illustrate the range of skills required. Each problem is followed by a step by step solution. By studying these examples, you'll develop a deeper understanding of problem solving strategies and common techniques.

Problem 1 – Basic Algebra

Question: If $3x + 5 = 20$, what is the value of x ?

1. A. 5
2. B. 4
3. C. 6
4. D. 7
5. E. 8

Solution:

1. Subtract 5 from both sides: $3x = 15$.
2. Divide by 3: $x = 5$.
3. Answer: A.

Key takeaway: Isolate the variable by performing inverse operations.

Problem 2 – Geometry: Circle Tangents

Question: A circle of radius 4 units is tangent to a square. The side length of the square is 10 units. How many points of tangency are there between the circle and the square?

1. A. 1
2. B. 2
3. C. 4
4. D. 8
5. E. 16

Solution:

1. Since the circle is tangent to the square, it touches the square at exactly one point on each side it is tangent to.
2. Because the circle's radius is less than half the side length (4 < 5), the circle is tangent to all four sides of the square.
3. Thus, there are four points of tangency.
4. Answer: C.

Problem 3 – Counting: Arrangements

Question: How many distinct 3 letter arrangements can be made from the letters A, B, C, and D if the letter A must appear in the second position?

1. A. 6
2. B. 8
3. C. 12
4. D. 24
5. E. 48

Solution:

1. The second position is fixed as A.
2. For the first position, we have 3 choices (B, C, or D).
3. For the third position, we have 2 remaining choices.
4. Multiply: $3 \times 2 = 6$.
5. Answer: A.

Problem 4 – Probability

Question: A bag contains 4 red marbles and 6 blue marbles. Two marbles are drawn at random without replacement. What is the probability that both marbles are the same color?

1. A. 0.4
2. B. 0.5
3. C. 0.6
4. D. 0.7
5. E. 0.8

Solution:

1. Probability both are red: $\frac{4}{10} \times \frac{3}{9} = \frac{12}{90} = \frac{2}{15}$.
2. Probability both are blue: $\frac{6}{10} \times \frac{5}{9} = \frac{30}{90} = \frac{1}{3}$.
3. Sum: $\frac{2}{15} + \frac{1}{3} = \frac{2}{15} + \frac{5}{15} = \frac{7}{15}$.
4. Convert to decimal: $(7 \div 15 \approx 0.4667)$, which rounds to 0.5.
5. Answer: B.

Problem 5 – Number Theory: Divisibility

Question: Which of the following numbers is divisible by 6?

- A. 123
- B. 234
- C. 345
- D. 456
- E. 567

Solution:

1. A number is divisible by 6 if it is divisible by both 2 and 3.
2. Check each candidate:
3. 123: not even !' not divisible by 2.
4. 234: even, sum of digits $2+3+4=9$!' divisible by 3. So 234 is divisible by 6.
5. 345: not even.
6. 456: even, sum $4+5+6=15$!' divisible by 3. Also divisible by 6.
7. 567: not even.

Problem 6 – Advanced Geometry: Triangle Ratios

Question: In triangle ABC, $AB = 10$, $AC = 12$, and the ratio of the lengths of the medians from vertices B and C is 3:4. What is the length of median AD?

1. A. 5
2. B. 6
3. C. 7
4. D. 8
5. E. 9

Solution:

1. The median from a vertex connects that vertex to the midpoint of the opposite side.
2. Let M be the midpoint of AC; then median from B is BM.
3. Similarly, let N be the midpoint of AB; median from C is CN.

4. Using Apollonius's theorem, the length of a median can be expressed as $(m_a = \frac{1}{2}\sqrt{2b^2 + 2c^2 - a^2})$.

5. Compute BM and CN:

6. BM: $(b = AC = 12, c = AB = 10, a = BC)$. But BC is unknown. However, we can use the ratio BM:CN = 3:4.

Note: This problem demonstrates that sometimes the AMC 8 encourages a “back of the envelope” approach rather than a full derivation.

Problem 7 – Combinatorics: Seating Arrangements

Question: Five friends are seated around a circular table. How many distinct seating arrangements are possible if two friends, Alice and Bob, must sit next to each other?

1. A. 12
2. B. 24
3. C. 36
4. D. 48
5. E. 60

Tags: #problems #solutions

